Final

The Canadian Mathematical Society presents

2025 Canadian Open Mathematics Challenge

Special thanks to our sponsor:

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Official Competition Booklet

Please completely fill the appropriate circles:

Will you be 19 years old <u>or under</u> on June 30, 2026? Ⓨ Ⓝ

Why we ask this: The COMC has an age limit for official participation matching next summer's IMO.

Are you a Canadian Citizen or Permanent Resident of Canada? Ⓨ Ⓝ

Why we ask this: We need to know which of our two primary divisions you qualify in: the Canadian Division or the Foreign/International Division.

Your email address: ____

Why we ask this: If your score on the COMC is high enough, we'll want to contact you for higher level invitation-only competitions such as the Canadian Mathematical Olympiad. ***Please print clearly!***

DO NOT PHOTOCOPY BLANK EXAMS
Each page has a unique pre-registered bar code corresponding to a particular student.

Question A1 (4 points)

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What is 200% of 2% of a third of 2025?

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question A2 (4 points)

How many integers between 10 and 500, inclusive, have their digits in strictly decreasing order? For example, 41 and 320 are such integers, but 441 and 230 are not.

Your solution:

Your final answer:

[A correct answer here earns full marks]

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Question A3 (4 points)

An integer, $n > 1$, is called *doubly squared* if n is a perfect square and the number of positive divisors of n (including 1 and itself) is a perfect square. Determine the smallest doubly squared integer greater than 1.

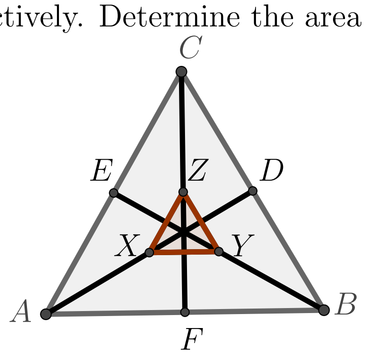
Your solution:

Your final answer:

[A correct answer here earns full marks]

Question A4 (4 points)

Let ABC be an equilateral triangle with area 80. Let D, E, and F be the midpoints of BC, CA, and AB, respectively. Then, let X, Y, and Z be the midpoints of AD, BE, and CF, respectively. Determine the area of triangle XYZ.



Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B1 (6 points)

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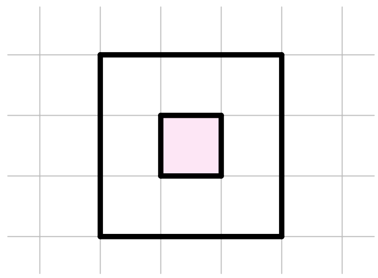
Min and Max each have a 4×4 grid of 16 unit squares. Each of them removes three of the unit squares in their grid, and then computes the perimeter of their resulting shape. What is the maximum possible difference in their answers?

Note: The *perimeter* of a shape is the sum of length of all the line segments that border the shape. For example, the following 3×3 square with the middle 1×1 square missing has the perimeter 16.



Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B2 (6 points)

Elizabeth keeps her photos in three boxes. There is some nonnegative integer k for which the first box contains $\frac{k}{5}$ of the total number of her photos, the second box contains 3/11 of the total number of her photos, and the third box contains 558 photos. How many photos does Elizabeth have in her collection?

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B3 (6 points)

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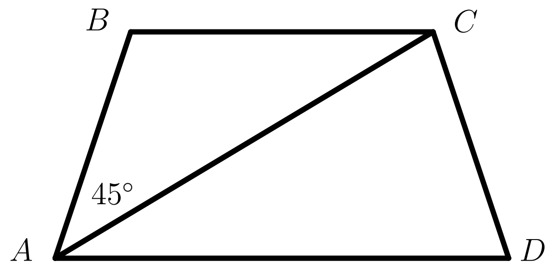
Let $ABCD$ be an isosceles trapezoid with sides $AB = CD$ and parallel sides $AD > BC$. It is given that $AC = AD = 10$ and $\angle BAC = 45°$. Determine the area of the trapezoid.



Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B4 (6 points)

Let $a_0, a_1, \ldots, a_{100}$ be fixed, pairwise distinct, and non-zero real numbers ($a_i \neq a_j$, $a_i \neq 0$, for all $0 \leq i \neq j \leq 100$). Consider the polynomial $p(x) = a_{100}x^{100} + a_{99}x^{99} + \cdots + a_1 x + a_0$. A polynomial $q(x)$ is obtained by choosing two distinct numbers among $a_0, a_1, \ldots, a_{100}$ uniformly at random, and swapping them in the expression of $p(x)$. The polynomials $p(x)$ and $q(x)$ are then graphed in the plane as functions of x. Determine the *expected* number of points of intersection in the graph of the two polynomials.

Here each intersection point is counted as one point regardless of its multiplicity.

The *expected value* of a random variable is the weighted average of the possible values the variable takes, weighted by their respective probabilities. For example, the expected number obtained from rolling a fair six-sided die is

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = \frac{7}{2}.$$

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question C1 (10 points)

For every integer $n \geq 2$, there is a *number system* with base n, where the integers are written using the digits $0, 1, \ldots, n-1$. For example, in the number system with base 16, also known as the *hexadecimal* system, the digits used are $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E$, and F, where the letters A, B, C, D, E, and F represent the numbers $10, 11, 12, 13, 14$, and 15, respectively.

A string of digits $\overline{abc}$ in the system with base n represents the positive integer $an^2 + bn^1 + cn^0$. (We will not write the bar over numbers written in the commonly used system with base 10.) For example, the number 314 in base 10 can be written as $\overline{13A}$ in the hexadecimal system, because $1 \cdot 16^2 + 3 \cdot 16^1 + 10 \cdot 16^0 = 314$. In general, a string of digits $\overline{a_k a_{k-1} \ldots a_0}$ in base n represents the positive integer $a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n^1 + a_0 n^0$.

For this problem, you may assume that every positive integer can be represented uniquely in base n for each $n \geq 2$.

(a) Write the base 10 number 2025 in base 5.

(b) Solve the equation $x^2 - \overline{20}x + \overline{AF} = 0$ written in the hexadecimal system. The answer should be given in the hexadecimal system.

(c) In what systems with base n is it true that $\overline{24}$ divides $\overline{2000}$?

Your solution:

You **must** show all your work.

Question C2 (10 points)

In mathematics, an anagram of a word is a rearrangement of its letters, including those that result in nonsensical words. A word is always an anagram of itself. For example, the word EAT has six anagrams: AET, ATE, EAT, ETA, TAE, and TEA.

(a) How many anagrams of the word $TENET$ start and end with T?

(b) How many anagrams of the word $YOOHOO$ start and end with O?

(c) Let N be the number of anagrams of the word $ABRACADABRA$ which start and end with different letters. Find, with proof, the largest prime number that divides N.

Your solution:

You **must** show all your work.

Question C3 (10 points)

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Call a positive integer N *shifty* if there exists an integer $d > 1$ such that the product $d \cdot N$ is made by removing some of the first digits from N and shifting them to the end (in the same order). For example, $N = 157894736842105263$ is a shifty number since

$$5 \cdot \mathbf{15}7894736842105263 = 7894736842105263\mathbf{15}.$$

(a) Prove that all shifty numbers are divisible by 3.

(b) Let N be a shifty number. Prove that there are infinitely many shifty numbers divisible by N.

(c) Find, with proof, all shifty numbers N less than 10^{15} such that shifting only the first digit to the end produces a multiple of N not equal to N itself.

Your solution:

You **must** show all your work.

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Question C4 (10 points)

Veronica plays a game against her 2025 students. First, each of the students points to exactly one other student in the room. Multiple people could point to the same student, and some students could have no one point to them. Veronica sees who pointed to whom.

Second, Veronica chooses N students to leave the room.

Third, Veronica assigns a number to each of the remaining $2025 - N$ students, under the condition that if one student is pointing to another, then they must receive the same number. If she assigns k distinct numbers to the students, then Veronica scores k points.

The students are trying to minimize the number of points Veronica scores, and she is trying to maximize the number of points she scores. Under optimal play from both Veronica and her students, what score does she receive when

(a) $N = 1$?

(b) $N = 100$?

(c) $N = 1000$?

Your solution:

You **must** show all your work.

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